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ABSTRACT

This document addresses issues raised during the Research on Teaching Mathematics Conference, examining them in view of the central tenets of the Institute for Research on Teaching at Michigan State University. It explains why teacher judgment is the focus of IRT research and discusses the concept of "the wisdom of the practitioner," noting its implications for research on teaching and raising questions about how practitioner wisdom might best be tapped. Two methods for studying teachers (process tracing, and description and reflection) are cited, and their advantages and disadvantages outlined. It is stated that information-processing research indicates teachers must simplify the teaching world to make it manageable. Researchers must conduct supplemental studies, using different methodologies, to determine how and why teachers choose certain modes of simplification rather than others. It is suggested that researchers can develop at least some teaching algorithms, which teachers would transfer into viable heuristics. Successful implementation of research requires effective dissemination of findings to teachers. A number of questions for future study are also presented. (Author/MP)

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Addendum to
Conference Series No. 3

INVESTIGATIONS OF MATHEMATICS TEACHING: A PERSPECTIVE AND CRITIQUE

Lee S. Shulman

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Preface

This paper was prepared as a conference summary for a three-day meeting on the study of teaching in mathematics, May 1-3, 1977, held at the Institute for Research on Teaching, Michigan State University. The major papers of the conference were presented by Perry Lanier, Michigan State University; Robert Davis, University of Illinois; Jeremy Kilpatrick, University of Georgia; Max Bell, University of Chicago; and John LeBlanc, Indiana University. Discussants for the papers were Robert Bridgham, Joseph Schwab, Edward Smith, Jere Brophy, Leo Landa, Jacob Kounin, Robert Hill, Judith Lanier, and Sharon Feiman.

INVESTIGATIONS OF MATHEMATICS TEACHING: A PERSPECTIVE AND CRITIQUE

Lee S. Shulman

Abstract

Shulman addresses issues raised during the Research-on-Teaching Mathematics Conference, examining them in view of the central tenets of the Institute for Research on Teaching. He explains why teacher judgment is the focus of IRT research and discusses the concept of "the wisdom of the practitioner," noting its implications for research on teaching and raising questions about how practitioner wisdom might best be tapped. Two methods for studying teachers--process tracing, and description and reflection--are cited, and their advantages and disadvantages outlined.

Shulman observes that information-processing research indicates teachers must simplify the teaching world to make it manageable. He proposes that researchers conduct supplemental studies, using different methodologies, to determine how and why teachers choose certain modes of simplification rather than others.

Shulman also suggests that researchers can develop at least some teaching algorithms, algorithms which he claims teachers--because of their wisdom--would transform into viable heuristics. He notes that successful implementation of research requires effective dissemination of findings, to teachers, and, finally, he offers a number of questions for future study.

About the Author

Lee S. Shulman is co-director of the Institute for Research on Teaching and professor of educational psychology and medical education at Michigan State University. He received his Ph.D. in educational psychology at the University of Chicago. Shulman is the author of many publications, including "The Psychology of School Subjects: A Premature Obituary?", which appeared in the 1974 issue of the Journal of Research in Science Teaching, and, Medical Problem Solving: An Analysis of Clinical Reasoning, which he co-authored with Arthur S. Elstein and Sarah A. Sprafka.

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Investigations of Mathematics Teaching; A Perspective and Critique*

Lee S. Shulman

The purpose of the conference, as outlined in the opening paper, was to identify important researchable questions concerning mathematics teaching. The organization of the conference followed closely the four commonplaces of education (Schwab, 1962): the subject matter, the learner, the teacher, and the environment. The element of teacher education was added as the clear mission of schools of education and institutes such as IRT.

We in the Institute have adopted a particular way of looking at teaching in general, and the teacher of mathematics in particular -- a view based primarily on human information-processing theory. We intended this conference as a means of improving the quality of research questions generated using information-processing theory, and as a means of identifying the limitations of this theory of the study of mathematics teaching.

The Judgment of Learning

An examination of any standard textbook in psychology or educational psychology reveals that learning, which is typically the central construct in educational psychology, has an interesting definition -- one which most educators have learned, over-learned, filed away, and probably forgotten. The standard definition is that learning is an inferred change in the learner which is indexed by observing changes in that learner's behavior not attributable to maturation. I find it

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*This paper was prepared as a conference Summary for the three-day Research-on-Teaching Mathematics Conference held at the Institute for Research on Teaching May 1-3, 1977.

remarkable that through the history of both educational psychology and the field of instruction we have systematically ignored the first part of that definition -- that learning is not merely a change in behavior, but rather is an inferred change of state in the learner. Our information-processing paradigm is, therefore, straightforward. If the business of teaching is understanding and influencing learning, then at the very center of teaching are processes of inference, processes of judgment and reasoning from behavior-as-cue to learning as an inferred state. Because we see this process of inference as essential, our work centers on the study of teacher judgment and perception.

An important point must be made about the relationship between the findings of educational research and the clarification and guidance of practice. The National Institute of Education held a conference (Rist & Anson, 1977) recently on education, social science, and the judicial process. The conference dealt with the relationship between educational and social science research and court decisions regarding school policies. Conference participants made frequent reference to the damning of social science research for not providing the clear-cut guidance that judges and juries believe should result from such research. Cohen and Weiss (1977) made the following observation in that regard:

> [There is] a misapprehension about the research process. The expectation of direct policy guidance from social research assumes that knowledge in the social sciences is convergent. It is expected that if knowledge improves, there will be more scientific agreement about the issues in question. But knowledge in the social sciences generally is not convergent. Rather than picturing the research process as scouts converging on a target, it might make more sense to picture it as outriders offering different visions of what passes them by. Multiplying the outriders tends to multiply the visions -- up to a point, of course. And sharpening their sight tends to refine their differences. (p.74)

Apparently, Cohen and Weiss had never read Schwab (1969) on the practical. Specialists in fields as diverse as curriculum and the study of social policy have recognized that, in principle, the successful research program need not lead to the algorithmization* of a field of practice. The role of algorithms in education and the relationships between algorithms and other means of organizing the work of a field will be discussed later. For now, it is enough to observe that research can serve teachers as it serves the courts -- by clarifying the complexities of nature, thereby making possible better grounded judgments, not by replacing judgment with ironclad rules.

Wisdom of the Practitioner

The most frequently recurring theme in this conference was reference to David Hawkins' (1966) observation that the best practice often exceeds the best theory in quite essential ways. Indeed, this belief in the "wisdom of the practitioner" is a central tenet of the IRT. A commitment to study the wisdom of practitioners, however, brings with it some interesting problems. First, what kinds of practitioner should be studied? Do we study Bob Davis, who is an outstanding practitioner of mathematics teaching? Or do we follow Leo Landa's admonition to avoid studying unusually good teachers because what they do has become so automatic that they cannot account for it, or because what makes them special is some unusual combination of genetic and special environmental conditions that cannot be reproduced

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*Algorithmization refers to formulating a set of explicit rules for practice to guide performance, such that "If situation x develops, then perform y."

in the rest of the teacher population?

Another related question is whether outstanding practitioners should be studied at all. We might well reformulate the question, asking not, "Who is the master teacher?" but rather "When is a teacher (or teaching) masterful?" Because there are certain days when even the most gifted teacher will do things badly and the least gifted teacher will hit the target, we really ought to be studying teachers per se, and the conditions under which they do their best work.

This observation raises other important questions. How do we index "their best work ?" During the course of this conference no one said anything good about standardized achievement tests as indicators of effective teaching. But what should be used as an indicator? Even if one finds short-term indicators one trusts, such as some sort of criterion referenced test, then one runs into what I will call the "Bell-Davis Paradox"; namely, there exist circumstances in mathematics teaching when successful achievement of short-term proximate goals is counterproductive in the long run.

Which of these -- short- or long-term outcomes -- should be used as criteria? It seems clear from the ethnographic studies of classrooms, ranging from Jackson (1968) or Smith and Geoffrey (1968) to IRT studies conducted by Janesick (1977), Yinger (1977), and Shroyer (1978), that teachers have already made that decision. They do not employ the distal, long-term goal represented by the end-of-year achievement test. Instead, teachers use the short-term process (student involvement, the quality of activity) as their criterion for how well things have gone. Ought we use as our criteria for masterful teaching those which wise practitioners already use as their functioning

criteria? This is a key question for research on teaching in general.

The problem of selecting examples of the master teacher or masterful teaching has been endemic to this kind of research.

Identifying masters is not difficult in chess and bridge, since these areas have consensual systems for determining who the masters are. In contrast, masters cannot be identified in medicine. What criteria would be used? Mortality rates? Peer review? Peer review was used in the Institute for Personality Assessment and Research at Berkeley by MacKinnon (1965) and Barron when they attempted to select outstanding mathematicians or advertising specialists. Though popular, peer review is nevertheless a complex process with its own built-in sources of distortion. We used peer nominations in our studies of physicians (Elstein, Shulman, & Sprafka, 1978) and became acutely aware of its limitations as a basis for selecting the master practitioner.

If the notion of the wisdom of the practitioner is to be taken seriously, then several implications for research must be examined. First, a practitioner cannot be simultaneously wise and mindless. In other words it makes no sense to derive data about the wisdom of practitioners in a way that reflects little respect for the significant contributions that the practitioner can make to the research. Likewise, it makes no sense to translate findings into programs of dissemination and training that treat the teacher as someone who lacks wisdom.

I was struck by Max Bell's observation that Jackson's (1968) Life in Classrooms often portrays the teacher as a non-thinking, non-reflective automaton. There is something ironic about that, because

I do not think that is what Jackson intended. The reader must recall something about the rhetoric of Jackson's argument in Life in Classrooms. The book is not only an ethnography of schooling, it is also a polemic, an aspect of his work frequently ignored. The objects of his attack were the programmed, computer-aided, and individually prescribed approaches to instruction that were, in the late 60s, the hallmarks of pedagogical technology in this country. Jackson argued that classrooms are typically fast-paced and frequently unpredictable. Because of these characteristics of classrooms, teachers are essentially, even necessarily, non-reflective.

It is a striking contrast to read Complexities of an Urban Classroom, a book written by Smith and Geoffrey (1968) in the same year as Life in Classrooms. The significant difference between the two studies is the research method used. Jackson studied three or four teachers as research subjects. Smith, a researcher, and Geoffrey, a teacher, were collaborators in their research. It is not really possible to collaborate with someone in research and then portray that person as mindless and non-reflective. Thus, the first dictum for research on the wisdom of the practitioner is that there can be no secrets in studies of teachers and teaching. Practitioners should know exactly what researchers are doing; they should share in thinking through how the study might best be conducted. With this type of collaboration, the concept of a research subject disappears. Teachers are not merely subjects. The phrase, "we're running subjects today," causes me great discomfort, especially when it comes out of my mouth; we all know the rat-maze tradition from which that phrase originates.

Methods of Research

The question of how to study the wisdom of practitioners is methodological as well as conceptual and ethical. As Perry Lanier noted, several approaches are available (Shulman & Elstein, 1975).

Process Tracing

Process tracing is introspective. Practitioners are asked, "Why did you do that?" It is not simple to introspect while performing tasks. One can play back video tapes of what was done and try to disentangle and reconstruct the parts of an experience. On the other hand, it is difficult to succeed with mere general retrospection. "You've been teaching for 30 years; tell me, how do you teach?" Researchers cannot ask this kind of question; they must try to make introspection as task-specific and immediate as possible. Introspection has long since outgrown some of the debilitating characteristics which encumbered it in the era of Titchener. Introspective research is done much more effectively now, and I think the kinds of research Simon, Newell, deGroot and others have conducted will withstand critical scrutiny.

There are, however, new things being learned about introspection; for example, the work conducted by Nisbett and Wilson (1977) regarding what can be learned about the cognitive functioning of people asked to introspect. Nisbett and Wilson argue that you cannot learn what individuals are really thinking as they do something. What you can learn is <u>their implicit theory of why reasonable persons like themselves would do</u> what they have done.

Even if this be the case, the fruits of introspective research

remain important. First, an implicit theory about why a reasonable person might act in a certain fashion may, in fact, be why they do act in that fashion. There may be an isomorphism between those two reports -- the normative and the ostensibly descriptive.

Second, the human being has a remarkable feedback system. Today's rationalization of yesterday's behavior may become the grounds for tomorrow's plan. If that is the case, then to understand these developing, personal, theories-in-use (Argyris, 1978) becomes a powerful finding, and a powerful tool for understanding not only why people do as they do, but perhaps the cognitive conditions under which they might be brought to do something else.

This leads to another recurring theme heard both in our formal sessions and informal discussions -- the call for more thorough descriptions of teaching. We often do not know enough about what is happening in classrooms and could profit from better descriptions. Description is important for research as well as for teacher preparation, the on-going and continuous development of teachers.

Description and Reflection

Schwab (1969), in his initial paper on the practical, describes the first stage of analysis for curriculum inquiries, not as discernment of purposes and goals, but rather as examination of what is really occurring in the instructional settings under investigation. This examination should be the starting point in the study of mathematics teaching, just as it is in so many other fields.

In a masterful paper, "On Problem Finding in Sociology," Merton (1959) makes precisely the same point about sociological research. He

argues that the first step in problem finding is carefully describing what the world is like, and that the worst kind of problem to pursue is one that never was.

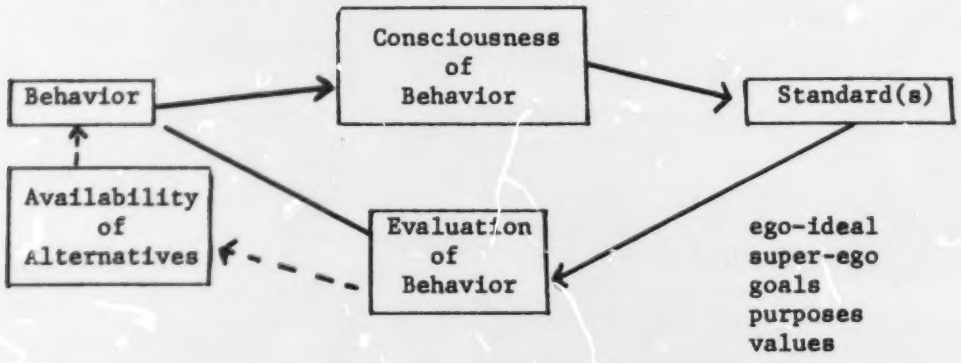
There was a fascinating exchange of letters to the editor in Science dealing with the history of operations analysis. Waddington (1974), one of England's leading operations analysts, described the research programs in which analysts worked through World War II for British Intelligence. The most important stage of operations analysis was developing clear and complete descriptions of the existing state of affairs. More often than not, once the "facts" were known, they did not need to know anything more to solve the problem. It was fairly obvious what had to be done.

This suggests not only why descriptive research is important in its own terms, but also relates to observations made at this conference by Brophy and by Feiman with regard to the role of certain kinds of feedback to teachers. They have both observed that often teachers need only a report of how they are actually behaving to effect a change in that behavior. Teachers need an opportunity to direct their attention to what is really happening in their classrooms. I suspect they also profit from suggested alternatives to their present functioning.

If we take this notion of the wisdom of the practitioner seriously these observations ought not to be surprising. Yet, they lead to another interesting research question. If a description is sufficient to lead to a change in teacher behavior, then teachers must be comparing that description against some implicit standard, expectation,

or other sense of what they ought to be doing. Where does that standard come from? How can it be influenced? Ought we to tamper with it at all?

The following figure represents the process of reflecting on one's own behavior as a facilitator of behavior change, with the intervening role of "standards"as mediators between consciousness of descriptions of one's own behavior and evaluations of it.



The Process of Reflecting on and Evaluating Behavior

Several points of intervention can be imagined in the system shown in the figure when the goal of intervention is the production of new behavior. First, one can directly manipulate behavior itself through control of contingencies of reinforcement. Second, the opportunity for self-observation or self-confrontation without explicitly attempting to influence the standards used to evaluate the appropriateness of that behavior appears to affect subsequent behavior.

A third source of behavior influence is critical review of the actor's standards themselves. Thus, environmental constraints can be employed to modify teacher behavior in particular directions, leaving

the teacher potentially dependent on such external forces in future situations calling for change. Researchers can also provide a variety of mirrors for teachers to view their own behavior, intentions, or effects, trusting in general processes of reflection and self-examination to trigger appropriate changes in their work.

Finally, researchers can look seriously at the underlying, typically implicit standards employed by teachers to evaluate their own work, similar to Rokeach's (1973) successful efforts to modify behavior toward minorities by confronting individuals with the discrepancies between their behavior and their values. The model presented in this figure is also consistent with Hammond's (1971) concept of Lens Model feedback (Brunswik, 1955), in which "outcome feedback," which informs the actor of his performance, is distinguished from "process feedback," which informs actors of the underlying policies that account for their performance.

One conference participant questioned whether education in mathematics will ever be improved as long as the kinds of text materials currently used in the mathematics teaching community continue to flourish. I see no alternative. Gene Glass and I engaged in an indirect debate several years ago at successive meetings of the National Association for Research in Science Teaching (Glass, 1972; Shulman, 1974). Glass argued that the solution to the problems of teaching lay in the improvement of the "materials and media of instruction." My argument was that the solution lay in the "minds and motives of school men." There will always be instructional materials of varying quality. Moreover, judgments of quality will frequently be situation-specific. The solution is to prepare teachers to sort through materials

and screen out what is inappropriate in their setting. I see no way of controlling what publishers in a free society will choose to produce. We must prepare teachers with the wisdom to make reasonable pedagogical judgments.

Convergence of Method

This focus on the practitioner is not only consistent with, but central to the approaches to research taken at the Institute. In the information-processing paradigm, one conducts research through modeling how human beings solve problems. When Newell and Simon (1972) want to learn about chess playing they do not conduct a logical analysis of chess; they study chess players and how they work. When they wish to understand the solving of crypt-arithmetic problems, they study people doing such problems. The focus of the research is on how individual problem solvers impose a problem space on a task environment, how they impose a simplifying perspective on the blooming, buzzing confusion of the world with which they must deal.

There is growing methodological convergence in the social sciences among approaches that share this commitment to people's perspectives as the explanatory construct in understanding why they behave as they do, though different social sciences use different terms to describe the construct. Information-processing psychology, which has become the mainstream of contemporary American psychology in learning and cognition, characterizes this internal state as a problem space. It attempts to explain how the intrinsic limitations of human information-processing capacities account for the producing of problem spaces and the ways in which problem-solving operations occur within them.

The problem space is also being studied by classroom ethnographers, albeit by other names. Participant-observation approaches in ethnography are employed because the researcher's goal is to internalize the perspective of the actors in the setting under investigation. Clifford Geertz (1973) has stated,

> Believing. . . that man is an animal suspended in webs of significance he himself has spun, I take culture to be those webs, and the analysis of it to be therefore not an experimental science in search of law but an interpretive one in search of meaning. (p.5)

Classroom ethnographers are searching for that set of shared meanings which permits people to function collectively in a particular setting. This is a collective form of problem space, jointly produced by the participants in the setting. Ethnomethodologists strive to understand the grammar of behavior, that implicit rule system governing everyday interaction, while symbolic interactionists like Goffman and others attempt to understand these perspectives in yet other terms.

If the convergence among these forms of inquiry is so clear, then why not simply choose any one to conduct research? It is hard enough to do information-processing research competently, so why not focus on that type of research and avoid the additional problems of these other forms of inquiry? The answer to that good question is that information-processing theory is insufficient. The information-processing psychologist attempts to explain human problem solving by relating the goals of the problem solver to the intrinsic constraints of human information processing. The psychologist focuses on the problems of information transfer between short-term and long-term memory which necessitate the simplification of a task environment

into a problem space.

Thus information-processing psychology explains why it is inevitable that simplification must take place. One can only handle four (plus or minus one) chunks of information in short-term memory. The older estimate of seven plus or minus two was unbelievably optimistic. Researchers learn two things from information-processing psychology: (1) how simply the situation must be defined in order to cope with it, and (2) to look to the task environment for the first clues to what the final problem space is going to be. What one cannot learn from information-processing psychology and can only learn from other forms of social science research is why and how human information processors choose particular constraints; why they respond to certain alternatives for defining the situation and not others.

The Chicago School of Sociology (Thomas and others) called these constraints the definition of the situation and argued they were critical to understanding why people behave the way they do. Additional modes of analysis such as those of the ethnographer are needed to address the ways in which human values, political pressures, the organization of classrooms and schools, the professional socialization of teachers, and a variety of other influences determine how the teacher selects a particular problem space from the almost infinite set of possible definitions of the situation and alternative simplification schemes.

Curriculum

The curriculum or subject matter is one of the elements of the task environment which exercises important influence on the teacher's

construction of the problem space for teaching. Some interesting things about curriculum have been learned over the years, perhaps well illustrated by returning to the Bell-Davis paradox. Restated, the paradox was that what is taught successfully in second grade can often have negative vertical transfer in that it inhibits learning something else in the seventh grade. As was discussed in the conference, it appeared this problem could be avoided if educators understood well enough what it was they wanted to teach across the total curriculum and thus preserved its continuity. I have an alternative hypothesis. I think we too long accepted what may be called Bruner's (1960) myth of the spiral curriculum. It was incorrect to assert that educators could devise a curriculum that would be, in principle, continuous and cumulative.

The myth of the spiral curriculum rested on two errors. (Those errors are stated in the topic sentences of Chapters 2 and 3, respectively, of the Process of Education.) The first error was the notion that if subject matters had a structure, then, in principle, a spiral curriculum would be possible -- not yet necessary, but possible. Bruner was apparently not impressed by how difficult it was for physicists, mathematicians, and biologists to agree on a structure for each of their subject matters. It was rarely suggested that perhaps the difficulty lay in there being no single structure for a discipline. Schwab brought that observation to the attention of the field about 15 years ago, but few understood its importance. In the Biological Science Curriculum Study (BSCS) program he and others argued for and constructed a curriculum in three versions, each reflecting an alternative structure of biology.

The second error we find in Bruner's (1960) discussion of curriculum (and it is related to the first one) is the assertion that any child can learn any topic at any time in an intellectually honest way. Unless you are prepared to defer all instruction until post-pubescence you must confront the inevitability that what you teach at earlier grades and in earlier years will be inconsistent with some things that you intend to teach later on. This is not a call for despair, however, but an important clue to some things that ought to be done both in curriculum development and teacher preparation.

In several of his papers, Bob Davis has talked about "torpedoing." Socrates talked about such a teaching method when he characterized himself as a "torpedo fish" who torpifies his listeners with his shocks -- a method whereby learners are lead down a primrose path toward a conclusion they know is correct, and then confronted with a counterexample. Davis argues that the most important kind of learning occurs when something you have previously learned or over-learned turns out to be inadequate. You then have to cope with the tension of the resulting disequilibrium and learn something at a higher level that incorporates both the earlier and later information. Imagine that multiplication is learned as repeated addition, which seems a reasonable way to think about it in second grade. By the time students begin multiplying fractions, they come to understand that multiplication cannot be simply repeated addition. Events like students learning to multiply fractions after they have already learned that multiplication is repeated addition become <u>strategic research sites</u> (Merton, 1959). These events become key moments to study in great detail in both teaching and learning because such discontinuities are inevitable

and fairly frequent. My guess is that although they are fairly frequent, these discontinities are difficult to overcome. For most children, at such a time, the most frequent form of learning is yet another "s-algorithm" which the child places next to the first one, learning a little story to account for the fact that they are both true, instead of learning a deeper more inclusive principle adequate to accommodate both events.

Curriculum discontinuity would become an important type of instruction to practice for those learning to teach. Those aspiring to become chess masters do not practice everything; they practice the particularly problematic aspects of chess, e.g., openings and end games. Teaching must also have particularly problematic aspects and a candidate for one of those is curricular discontinuity.

Teaching and Learning as Information Processing

It is important to think about both the learner and the teacher of mathematics as information processors. Perhaps it is even possible to dream of a unified information-processing theory of teaching and learning. During the conference someone humorously made reference to the Bionic Teacher, but as I thought about the relationship among teaching, learning, and curriculum, including the discontinuity problem noted by Davis and Bell, it struck me that pupils are as much bionic as teachers. Pupils are not blank slates. In a very real sense, educators take curriculum and teach it to pupils over time to create cognitive artifacts that grow in those pupils' heads. They thus create bionic pupils. By building little subroutines and teaching them to pupils, they influence those important individual differences

youngsters bring to subsequent learning settings. What is learned, however, is not a simple copy of what is taught.

One of the things confirmed from recent information-processing research on the learning of mathematics is that the learner is active. Educators may teach pupils algorithmic routines, but the children modify them to their own purposes. Groen and Resnick (1977), for example, taught youngsters computational algorithms for subtraction. However, their pupils reworked those algorithms into personally invented heuristics that simplified and in many cases improved on the algorithms. The same thing happened to one of our doctoral candidates in Science Education, but he attributed it to a failure in his teaching. That is, the candidate felt his experimental treatment had not worked because the elaborate strategies he had taught pupils for sorting or classifying rocks had not been employed in precisely the way that he had taught them. He considered that a disconfirmation of a number of his hypotheses. Several of us on his committee suggested that he consider it as evidence for the wisdom of his learners, and as further support for Resnick and Groen's observation that teaching algorithms does not necessarily lead to learning to apply those same algorithms.

This discussion is reminiscent of debates held 10 years ago at the height of the learning-by-discovery controversy. The debates centered on questions such as, "Can you teach in a guided manner and still have pupils learn to discover? Can you teach by discovery and still have pupils learn rules?" These questions once again have important implications -- for studies in the learning of mathematics, for understanding the outcomes of teaching, and for thinking about the

preparation of mathematics teachers.

The reason we at IRT are so interested in a parallel information-processing theory of teaching and learning is that what we assume to be true of learners should also be true of teachers. It is a very simple notion, yet ignoring it has led to a good deal of nonsense in education. The very same people who argued that any pupils could learn anything in some intellectually honest form then turned around and felt perfectly comfortable attempting to develop teacher-proof curricula. The pupils could learn anything, but the teachers were assumed to be incapable of adapting. By the same token, I suggest that educators think about the relationship between the teaching of algorithms and their transformation into heuristics in the minds of youngsters, and the implications that relationship might have for teaching pedagogical algorithms to teachers and their transformation into teaching strategies.

I shall state a hypothesis and try to develop it. While I am not as optimistic as Landa or Brophy about the likelihood that we will find many algorithms for teaching, I suspect that we will find some. I suspect that the algorithms for teaching will be no more exact than the algorithms in medicine. In fact, what we call algorithms in teaching are not like algorithms in mathematics, since they do not guarantee the desired outcome. They are really more like what Landa has dubbed quasi-algorithms because the solutions they provide have a confidence interval around them. That is the best one can do in the practical arts of medicine and teaching.

We will find such algorithms, and the wisdom of the practitioner will guarantee that they will not be employed mindlessly. Just as the algorithms for doing mathematics are transformed intelligently into heuristics in the minds of children learning mathematics, so

algorithms for teaching will either be transformed into heuristics or be controlled by higher order cognitive strategies in the minds and hands of teachers. I suggest that investigation of this phenomenon can be another strategic research site.

I am reminded that the Merck Manual, so often used as the model for the compendium of medical algorithms, is rarely used algorithmically by physicians. My colleagues and I (Elstein, Shulman, & Sprafka, 1978) have studied dozens of physicians and neither during formal nor informal observations have I seen physicians open a book, trace a set of symptoms, and follow a decision tree to a diagnosis or treatment plan. They simply do not operate that way. In part, this is because algorithms are frequently based on classic cases. The classics in medicine and education, as in literature, are most readily found in the library, not in the clinic or classroom. Thus, the algorithm, while useful, must be employed heuristically by the professional.

Scripts

How does one go from the algorithmic to the heuristic? Jed Lewis has acquainted some of us with "scripts" as a recent concept in the simulation of thinking (Schank & Abelson, 1977). This led us to think about the more traditional notion of scripts. One idea discussed was whether educators should secure some scripts of a good teacher's performances and have a new teacher follow those scripts. I thought about how people learn chess or bridge and one of the things they do is to replay master's games. They follow step by step the strategies and stages that someone else has taken, which are accompanied

by a discussion of why those steps were taken and what alternatives were considered.

In an interview a number of years ago with the late James Jones after he wrote the best-selling novel, From Here to Eternity, Jones was asked how he learned to write. He told how he was trained by a teacher of writing in downstate Illinois who had her students spend day after day copying examples of great literature in longhand. Although that sounded like nonsense to me, I wondered whether there was not something to learn from it. Might this process not be related to the transformation of algorithms, decision trees, or other prescriptive models into heuristic adaptations by the learner?

One of the inexorable blessings of human cognition is the process of invention and construction. We do not merely imitate; we always construct. If we construct on the basis of systematic and well thought-out models, our adaptations and constructions are likely to be more intelligent.

Dissemination

The question of dissemination has also been raised a number of times. The obvious questions here have to do with what kind of knowledge one wants to disseminate. If there are laws of learning and teaching, then it makes sense to disseminate those. If what is really important is helping teachers understand what they are doing, then it is more important to disseminate processes of reflection and self-examination than the laws of learning. I have learned a great deal in trying to report to practitioners the results of my own and others' research both in the doing of medicine and the doing of teaching.

It is very simple to report to educational researchers about how physicians practice medicine or about how teachers teach. Researchers are not concerned with how well the report you had made fits with the phenomenology of the practitioner; they are concerned with whether you calculated your degrees of freedom correctly. It is a different kind of test, however, to relate research results to practitioners. Reporting research back to the practitioners, who are themselves the objects of study, becomes an interesting test of what one has discovered that is worth disseminating and what constitutes useful, effective, and fruitful dissemination.

There is a small difference between what is trivially obvious to tell teachers and what is surprisingly plausible. There are some things researchers can say to teachers (or practitioners of any kind) that are so obvious to them and have been part of the profession's lore for so long that they do nothing to further teachers' understanding. When I have spoken to physicians about our findings concerning the nature of medical diagnostic work (Elstein, Shulman, & Sprafka, 1978), the typical reaction has been, first a look of puzzlement and then a kind of sheepish smile -- a shake of the head -- and a look of amused agreement and recognition. It is as if they were saying, "I hadn't thought about it that way, but, in effect, you've laid a framework on what I do and, by God, it makes sense."

Similarly, there is work in research on teaching, including the best of Kounin, Brophy, or Jackson, that is consistent with the phenomenology of the teacher, while not immediately obvious to him/her.

I have a sense that this is something akin to what Piaget discusses as the proper equilibrium between assimilation and accommodation,

an experience or communication that stretches the learner enough to require some modification of category, and yet is not so remote from the learner to preclude the possibility of accommodation. This is also reminiscent of the psychoanalyst's art in producing an interpretation of what the patient or client has said or done which is timed so the interpretation is neither gratuitous ("See how well I can relate your problems to psychoanalytic theory") nor simplemindedly non-direct ("I think what you said is you're angry"). Rather, it is an interpretation that brings insight and greater wisdom to the client in the interaction.

I believe that such feedback is a form of dissemination we could derive from studying the wisdom of the practitioner and that could in turn be used to influence the wisdom of other practitioners. Such knowledge would provide practitioners with more and better ways of understanding what it is they do. Researchers have the capability for producing such insight from information-processing research, research on judgment, and research in the ethnographic tradition. Tools are available for representing those forms of feedback and presenting them to learners. These tools range from the notion of process feedback in the Brunswik and Hammond Lens Model tradition and methods of "boot strapping" in the policy-capturing tradition (Shulman & Elstein, 1975), to the use of videotape feedback for reflection as in the work of Kagan (1967). These are ways of feeding back alternative representations of what occurs in people's heads to help those people studied do better in what they do.

This discussion suggests some tests that might be used to measure the value of findings from research on teaching. One of the tests

would be whether those findings take a form that helps teachers grow. Put another way, are the conclusions researchers have come to sensible, usable, and enlightening as perceived by the practitioners? If the object of research on teaching is less law-giving and more the development of the practitioner's intuitions and skills, then the final test must be whether in fact this understanding does improve those practical clinical abilities.

I refuse to be drawn into the conflict between intuitions and rationality -- between the art and the science -- because I see them blended and intertwined in the work of the teacher. Therefore, I feel compelled to use all the talents available in the educational research community to try to make what is lawful in teaching into explicit rules and also to characterize as clearly as possible what is intuitive and how those intuitions might be enhanced. Just the fact that intuition is recognized as central to the endeavor does not mean that researchers must forego all scientific tools and become totally intuitive. I am reminded of Clifford Geertz's (1973) observation:

> I have never been impressed by the argument that, as complete objectivity is impossible in these matters (as, of course, it is) one might as well let one's sentiments run loose. As Robert Solow has remarked, that is like saying that as a perfectly aseptic environment is impossible, one might as well conduct surgery in a sewer. (p.30)

I think educational researchers can develop appropriate human blends of the intuitive and the systematic and need not choose up sides.

Questions for Future Inquiry

Rather than summarize what has taken place, I have used the discussion of the last two days as a stimulus for thinking about a number

of things with which we in the Institute are concerned. Let me conclude by reiterating a series of thoughts and questions that might serve as a partial agenda for future discussion. First, concerning the usefulness of the subject area mathematics as an appropriate organizer for research on teaching, I was intrigued by Davis's claims, echoed to some extent by Bridgham, regarding the broad waves cast by the learning of mathematics. One does not only learn mathematics but also additional kinds of learning that spread to other areas. Of course, such claims are made in nearly every subject. People in the reading field say that when students learn reading well, they can be learning any of the content areas.

If there is truth to these claims, and I suspect there is, perhaps the discipline is an inappropriate unit of analysis. I recall with some irony the story of Abraham Flexner (1940/1960), who inspired the "Flexner Curriculum," a curriculum in medical schools that deals with each of the basic disciplines separately. In that curriculum, a medical student learnes each of the disciplines separately before being allowed into the clinical years. That same Abraham Flexner created the curriculum of the Lincoln School of Teachers College, Columbia, which expressly eschewed organization by separate disciplines. The curriculum of the Lincoln School was organized around topics or problems, like ships and ship building. (That program and the second-grade curriculum on ship building was discussed in Lawrence Cremin's (1961) Transformation of the School.) Clearly, Flexner believed that in certain areas, such as elementary education, organization in terms

of the separate subjects made less sense than they did in other areas, such as professional education. There might be a message here.

Second, there was a real tension between Bob Davis on the one hand, and John LeBlanc and Max Bell on the other hand, with respect to what it is that should be the goal of mathematics teaching and the preparation of mathematics teachers. I felt that LeBlanc and Bell were using as their starting point the million plus teachers currently teaching mathematics, while Davis was using an image of how mathematics ought to be taught. I think that is one issue we must confront.

Third, there is a set of questions that I raised earlier about the master teacher. Who should be studied for the development of teaching algorithms or normative models? If masters should be studied, how should they be chosen? By studying teachers who are best by consensus, how does one avoid the danger of non-reproducible talents? If it is masterful teaching among teachers in the normal range that is to be studied, what criteria should be employed -- long range or short range; cognitive learning or student involvement; achievement or management?

Fourth, I think we ought to examine the relationships between heuristics and algorithms both for teaching and for learning. Researchers are interested in heuristics and algorithms both as modes of instruction and as outcomes. Are they best subsumed under one another or are they mutually exclusive? As I said before, this issue is reminiscent of earlier arguments concerning learning by discovery, and whether it should be looked at as a mode of teaching, as a process occurring in the learner, or as an outcome of instruction. I have a feeling that we are back into that bind again in talking

about heuristics and algorithms, and I hope that this bind will not last as long as the one in discovery learning.

Fifth, are the experiments with scripts or computerized preceptors as in the IRT CLIPIR (Clinical Information Processing in Reading) project worth pursuing in the training of teachers? In these experiments teachers are given algorithmic training in the hope they will become heuristically flexible and diversified teachers.

Sixth, if pupil learning of mathematics is an information process, as is teaching, how can researchers learn to connect the two? What cues or indicators can teachers learn to use to index the underlying learning or mislearning of pupils? What might the teachers in Erlwanger's (1975) studies have picked up two years earlier to prevent or divert the "weird" learning that those pupils achieved in fourth grade? I think this becomes a central question for both the preparation of teachers and the study of teaching.

And finally, what is the appropriate ratio of resource allocation between in research and in teacher preparation between the materials and media of instruction on the one hand -- the products, the curriculum materials, the new programs, the algorithms, the decision trees -- and the minds and motives of teachers, on the other? Training the teacher clinically is a different investment than producing better materials.

In the 60s and early 70s the payoff was in materials. We are told now, however, that neither NSF, NIE, nor OE any longer intends to support the development of materials. I find it painfully ironic that USMES lost its funding, not because it did not work, but because it was charged with being more a philosophy of teaching than a "real

curriculum." I am left puzzled. If they won't buy products and they won't buy a process, what do we have to sell? What will be the result of our research and development efforts if it is neither of those?

Concluding, I apologize to those whose words I have distorted and even more to those to whom I have been unable to make reference in these remarks. I hope we will be able to use these remarks as an incomplete summary of our proceedings, but, more important, as a stimulus for further summaries and examinations of the deliberations we have had.

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